THE INFLUENCE OF CURING RESIDUAL STRESS ON THE STRENGTH AND FRACTURE ROTATING SPEED OF COMPOSITE DISC

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Abstract

In this paper, the influence of thermal residual stress on strength and fracture rotating speed of composite disc is studied and analyzed using finite difference method and laminated plate theory for two types of reinforced composite discs (radial fiber reinforced disc and circumferential fiber reinforced disc). As a result the thermal residual stress will reduce radial and tangential stresses in radial fiber reinforced disc, while it shall increase radial stresses and decrease tangential stresses in circumferential fiber reinforced disc. The existing of residual stresses in composite disc will leads to initiation of crack and begins to propagation near the inner diameter compared to case when neglecting residual stress for cases taken in analysis. It is also verified that the finite difference method is a good tool for stress analysis of composite disc under residual stress effec

دراسة تأثير الأجهادات المتبقية على مقاومة وسرعة الأنكسار للأقراص المصنوعة من المواد المركبة.

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في هذا البحث، تم تحليل ودراسة تأثير ألاجهادات المتبقية الحرارية على مقاومة وسرعة الانكسار للأقراص الدوارة المصنوعة من المواد المركبة باستخدام طريقة الفروقات المحددة ونظرية الألواح الرقائقية لنوعين من الأقراص الدوارة المقواة بالألياف (قرص مقوى بألياف قطرية وقرص مقوى بألياف محيطيه). أن ألاجهادات المتبقية سوف تسبب انخفاض في قيمة ألاجهادات القطرية والمحيطية للقرص المقوى بألياف قطرية، بينما تسبب زيادة ألاجهادات القطرية وتناقص ألاجهادات المحيطية في الأقراص المقواة بألياف محيطيه. وجود ألاجهادات المتبقية داخل القرص سوف يؤدي إلى نشوء الشق وانتشاره بالقرب من القطر الداخلي للقرص مقارنة بحالة عدم وجود ألاجهادات المتبقية للحالات التي تم دراستها. كما تم تحقيق بان طريقة الفروقات المحددة تعطي نتائج جيدة عند تحليل الأقراص الدوارة تحت تأثير ألاجهادات المتبقية.

1- Introduction

There are wide range of residual stresses in the composite materials, in general there are two types of residual stresses according to reason of emergences. First, when the fiber and mixture have different coefficient of thermal expansion, residual stresses arise upon cooling from the processing temperature. Second, residual stresses result from pressing and primary process (thermal process) to construct the primary layer of fiber reinforced matrix which is called curing process [1,2].

In recent year, the use of rotating disc manufacture from fiber reinforced material increased due to low cost of fiber. The study of bearing rotating speed and rotating strength is very important in desgin. Although there are many method to analyses rotating strength , Hoofman's fracture criterion is one of the most use since it easy in programing [3].

There are many studies on fiber reinforced disc most of them focused on optimal lamination and residual stresses result form different coeficient of expansion of fiber and matrix. Byon and Uemuru [4], studied optimal lamination in fiber reinforced flywheel. Marshall and Evans [5] use analytical method to pridect the influence of residual stresses on the toughness of brittle material. Budiansly and Hutachison [6] developed a relation connecting between critical residual stress and J-Integral approach in the brittle matrix composite materials.

In this paper the finite difference method and laminated plate theory accompanied by the residual stress that result from curing process are analysed for a disc reinforced by radially fiber and for a disc reinforced by circumferential fiber.

2- Kinds of Reinforced Discs and Elastic Constants

As studied earlier, two types of reinforced disc are taken for the analysis (radially fiber-reinforced disc and circumferentially fiber reinforced disc). The elastic constants for each type are given by the laminated plate theory (LPT) as follow [3]:

Define the subscribes “L” and “T” to refer to the longitudinal and transverse direction to fiber. The subscripts “ r “ and “θ” refer to the radial and circumferential directions.

Let:

E: Young’s modulus.

υ: Poisson’s ratio.

α : Coefficient of thermal expansion.

G: Modulus of rigidity.

Then,

A) For radially fiber reinforced disc shown in Fig.1-A, the elastic constants are:

$$E_r = E_l \ , \quad E_\theta = E_t$$

$$\upsilon_r = \upsilon_l \ , \quad \upsilon_\theta = \upsilon_t$$

$$\alpha_r = \alpha_l \ , \quad \alpha_\theta = \alpha_t$$

B) For circumferential fiber reinforced disc shown in Fig.1-B, the elastic constants are:

$$E_r = E_t \ , \quad E_\theta = E_l$$

$$\upsilon_r = \upsilon_t \ , \quad \upsilon_\theta = \upsilon_l$$

$$\alpha_r = \alpha_t \ , \quad \alpha_\theta = \alpha_l$$

Table (1) show the elastic constant of carbon fiber reinforced epoxy resin [3].

Table 1. Material constants for carbon fiber reinforced disc.

Quantity	Constants
E_L	139.4 Gpa
E_T	8.33 Gpa
G_{LT}	4.84 Gpa
ν_L	0.316
ν_T	0.0165
α_L	$0.372*10^{-7} / {}^oC$
α_T	$0.365*10^{-4} / {}^oC$
ρ	$1780 \ kg/m^3$

3 – Developing of Governing Equations

The radial and tangential strains are calculated using the relations [6]:

$$\varepsilon_r = \frac{du}{dr} \qquad \ldots (1)$$

$$\varepsilon_\theta = \frac{u}{r}$$

Where,

u: Radial displacement.

r: radius of disc.

The stress components are given by Hook's law[6]:

$$\sigma_r = \frac{E_r}{(1-\upsilon_r\upsilon_\theta)}[\varepsilon_r + \upsilon_\theta\varepsilon_\theta - (\upsilon_\theta\alpha_\theta + \alpha_r)\Delta T] \qquad \ldots (2)$$

$$\sigma_\theta = \frac{E_\theta}{(1-\upsilon_r\upsilon_\theta)}[\varepsilon_\theta + \upsilon_r\varepsilon_r - (\upsilon_r\alpha_r + \alpha_\theta)\Delta T]$$

Where,

ΔT : Temperature drop during cure process.

The equilibrium equation for the disc with uniform thickness is [7]:

$$\frac{d(\sigma_r)}{dr} - \sigma_\theta + \rho\omega^2 r^2 = 0 \quad \ldots (3)$$

Where,

ρ : Specific density.

ω : Angular velocity of disc in (rad/s).

Under the boundary conditions:

$\sigma_r = 0$ at $r = r_i$ and at $r = r_o$

Where,

r_i : Inner raduis of disc

r_o : Outer radius of disc.

The radial displacement (u) can be analytically given by introducing eq.1 and eq.2 into eq.3 as follows:

$$\left.\begin{array}{l} \frac{u}{r_2}=\frac{\rho r_2\omega^2(1-\upsilon_r\upsilon_\theta)*\bar{r}^3(3+\upsilon_\theta)}{E_r(\eta^2-9)*(\lambda^{\eta-1}-\lambda^{-\eta-1})} \\ *\{-\frac{(\lambda^2-\lambda^{-\eta-1})}{(\mu+\upsilon_\theta)}*\bar{r}^{(\eta-3)} \\ +\frac{(\lambda^2-\lambda^{\eta-1})}{(\upsilon_\theta-\eta)}*\bar{r}^{(-\eta-3)}+ \\ \frac{(\lambda^{\eta-1}-\lambda^{-\eta-1})}{(3+\upsilon_\theta)}\}+\frac{\Delta T(\alpha_\theta-\alpha_r)}{(1-\eta^2)}* \\ \{\frac{(1-\lambda^{-\eta-1})}{(\lambda^{\eta-1}-\lambda^{-\eta-1})}(\eta-\upsilon_\theta)*\bar{r}^\eta+ \\ \frac{(1-\lambda^{\eta-1})}{(\lambda^{\eta-1}-\lambda^{-\eta-1})}*(\eta+\upsilon_\theta)*\bar{r}^\eta\} \\ +\{\frac{\alpha_\theta(\upsilon_\theta-\eta^2)-\alpha_r(\upsilon_\theta-1)}{(\alpha_\theta-\alpha_r)}*\bar{r}\} \end{array}\right\} \quad \ldots(4)$$

Where,

$$\bar{r}=\frac{r}{r_o}$$

$$\lambda=\frac{r_i}{r_o}$$

$$\eta^2=\frac{E_\theta}{E_r}$$

4- Finite Difference Approach (FDM)

The finite element method is used in analysis of residual stresses in composite disc because of that require very fine mesh radial points and alarge number of repeated operation to calculation of radial and tangential stresses. The achieve this operation by using finite element method is very complicated in programming software; therefore use finite diference method.

The finite difference method (FDM) is a classical and a straight forward numerical method,the analysis consists of transforming the continuous domain of the state vaiables to a network or mesh of discrete pointes. The equilibrium and compatibilty equations of the rotating disc are given by the following equations [8]:

$$\frac{d}{dr}(r\sigma_r)-\sigma_\theta+\rho\omega^2r^2=0 \quad \ldots(5)$$

$$\frac{d}{dr}\varepsilon_\theta+\frac{\varepsilon_\theta-\varepsilon_r}{r}=0 \quad \ldots(6)$$

Substituting eq.1 and eq.2 to eq.5 get

$$\frac{d}{dr}(\frac{\sigma_\theta}{E_\theta}-\delta\sigma_r)+\{\frac{1}{r}(\frac{1}{E_\theta}+\delta)\sigma_\theta-(\frac{1}{E_r}+\delta)\sigma_r\}=0 \quad \ldots(7)$$

Where,

$$\delta=\frac{\nu_r}{E_r}$$

Solve eq.7 with eq.6, then the equation for radial and tangential stresses at discrete radial points becomes as followes [9]:

$$\frac{r_{i+1}}{r_{i+1}-r_i}\sigma_{r_{i+1}} - \frac{r_i}{r_{i+1}-r_i}\sigma_{r_i} \quad \ldots (8)$$
$$-\frac{\sigma_{\theta_{i+1}}-\sigma_{\theta_i}}{2} = \frac{-\omega^2\rho}{2}(r_{i+1}^{\ 2}-r_i^{\ 2})$$

$$\left.\begin{array}{l} [\frac{\upsilon_r}{E_r(r_{i+1}-r_i)}+\frac{1+\upsilon_r}{2r_{i+1}E_r}]\sigma_{r_{i+1}} \\ +[\frac{-\upsilon_r}{E_r(r_{i+1}-r_i)}+\frac{1+\upsilon_r}{2E_r r_i}]\sigma_{r_i} + \\ [\frac{-1}{E_\theta(r_{i+1}-r_i)}-\frac{1+\upsilon_\theta}{2r_{i+1}E_\theta}]\sigma_{\theta_{i+1}} \\ +[\frac{1}{E_\theta(r_{i+1}-r_i)}-\frac{1+\upsilon_\theta}{2r_i E_\theta}]\sigma_{\theta_I} \\ -\frac{\Delta T}{2}[\frac{1}{r_{i+1}}+\frac{1}{r_i}](\alpha_\theta-\alpha_r)=0 \end{array}\right\} \ldots (9)$$

i: represent point number of node in finite diference meshes.

In this paper 100 nodes hase been taken for analysis

5 – Fracture Rotating Speed

Hoffman's fracture criterion is adapted to predict the cracking rotating speed of two types of disc used in analysis, this fracture criterion for plane stress state is given by the following equation [9]:

$$\frac{x^2\sigma^2{}_\theta - X\sigma_r\sigma_\theta}{F_{\theta t}F_{\theta c}}+\frac{x^2\sigma^2{}_\theta}{F_{rt}F_{rc}}+ \quad \ldots (10)$$
$$\frac{F_{\theta c}-F_{\theta t}}{F_{rt}F_{rc}}x\sigma_\theta+\frac{F_{rc}-F_{rt}}{F_{\theta t}F_{\theta c}}x\sigma_r=1$$

Where,

σ_r : Radial stress.

σ_θ : Circumferential stress.

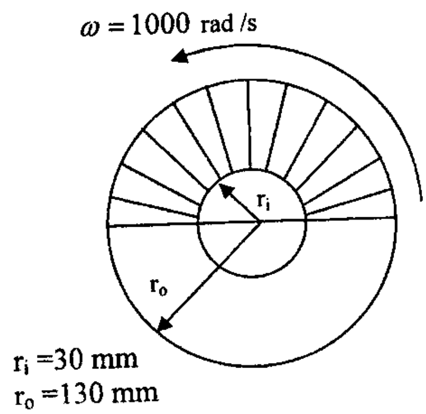
The notation F denotes the ultimate strength and suffix " t " and " c " means tension and compression respectively. Equation (7) is a quadratic equation of (x) , the rotating speed at which fracture will occur at point (x_o) in the disc is given by [10]:

Fracture rotating speed =

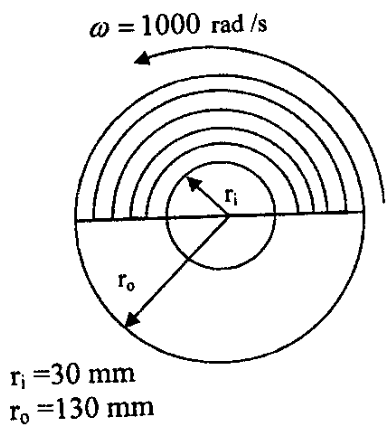
$$\sqrt{x_o} * \omega * \frac{60}{2\pi} \text{ r.p.m.} \quad \ldots (11)$$

After calculating fracture rotating speed for radial point's (X_o), plot a curve between fracture rotating speed and radius , the crack will initiate when the rotating speed of it reaches minimum value of the curve [11].

Fig.1 show two types of reinforced disc made from carbon fiber reinforced material [9]. Table (2) shows the constants of the failure conditions i.e. the ultimate strength in tension and compression for carbon fiber reinforced disc which is called (Hoffman's constant). This can be obtained it from tension and compression test of disc material [9].



A – Radial fiber reinforced disc.



B- Circumferential fiber reinforced disc.

Fig1. Types of reinforced discs.

Table 2 . Hoffman's constants for carbon fiber reinforced disc [9].

Quantity	Constants
F_{rt}	40 MN / m^2
$F_{\theta t}$	778 MN / m^2
F_{rc}	534 MN / m^2
$F_{\theta c}$	112 MN / m^2

6- Results and Discussion

As illustrated in Figs.2, 3, 4, and 5 there are closeness between results obtained from finite difference method and laminated plate theory. Also the finite difference method give lower values compared to laminated plate theory.

From Fig.2 the residual stress will reduce radial stresses and oscillate it from negative to positive values compared to case when neglecting residual stress. While in Fig.3, the tangential stresses will drop from maximum value to smallest value at inner radius as a result of existing residual stress.

From Fig.4 and Fig.5 for circumferential fiber reinforced disc, the residual stress will increase radial stresses to large values compared to that without residual stress, while the tangential stress will decrease.

Fig.6 and Fig.7 illustrated the effect of curing residual stress on fracture rotating speed of radial fiber reinforced disc and circumferential fiber reinforced disc. For radial fiber reinforced disc, the fracture rotating speed has minimum value at a radius (62 mm), so the crack will initiate at this point. For circumferential fiber reinforced disc the fracture rotating speed has minimum value at radius (55 mm) so the crack will initiate at this point.

When neclecting residal stress, the fracture rotating speed for radial fiber reinforced disc has minimum value at radius (100mm), while for circumferential fiber reinforced disc the fractur rotating speed has minimum value at a radius (91 mm).

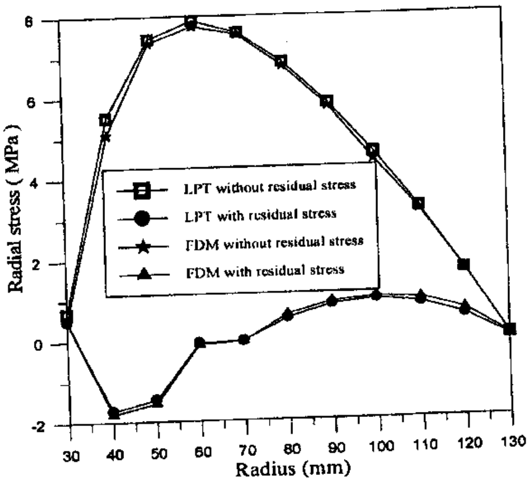


Fig.2 Radial stress distribution for radial fiber reinforced disc

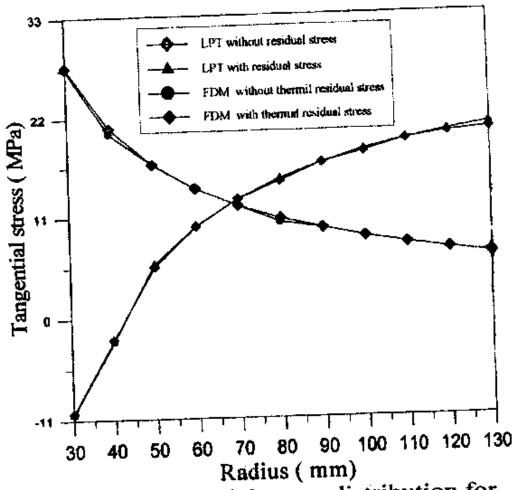


Fig.3 Tangential stress distribution for radial fiber reinforced disc.

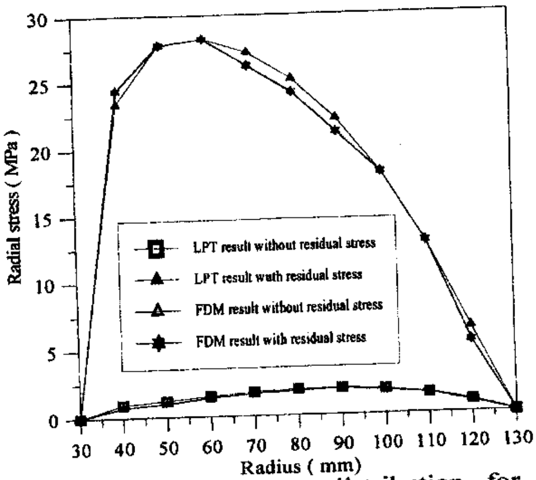


Fig.4 Radial stress distribution for circumferential fiber reinforced disc.

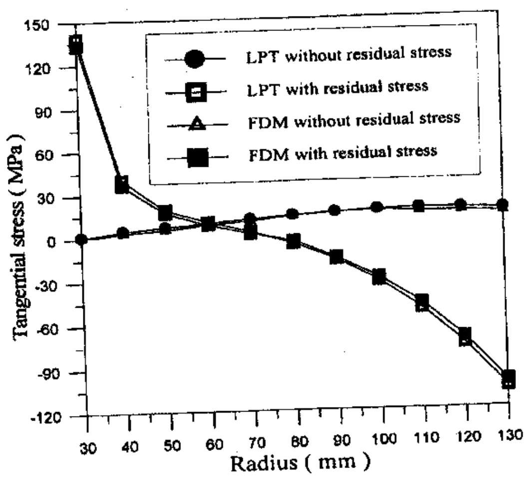


Fig.5 Tanential stress distribution for circumferential fiber reinforced disc.

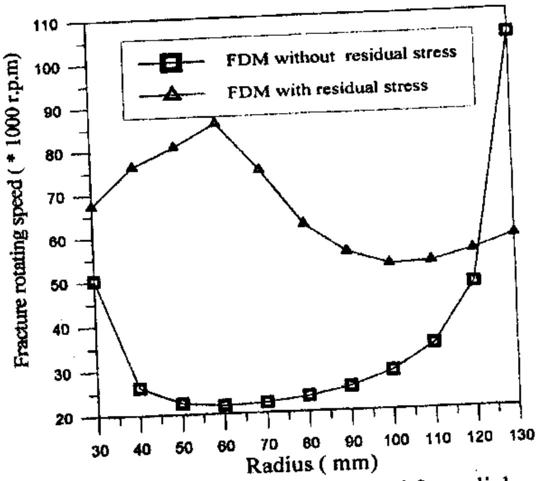


Fig.6 Fracture rotating speed for radial fiber reinforced disc

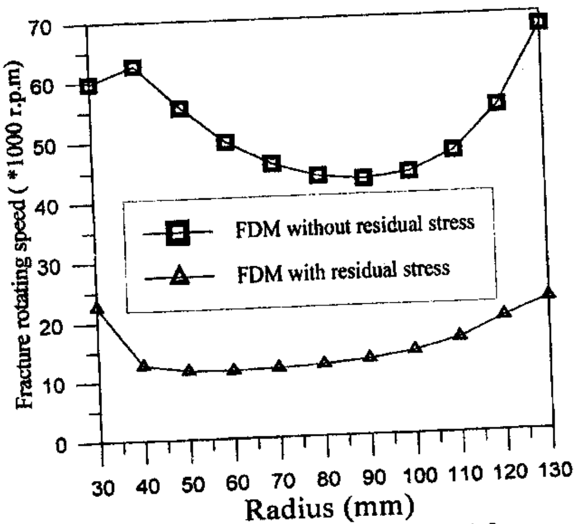


Fig.7 Fracture rotating speed for circumferential fiber reinforced disc.

7- Conclusions

In this paper, the influence of thermal residual stresses results form curing process on the strength and fracture rotating speed of composite disc are carried out using finite difference method (FDM) and laminated plate theory (LPT) for two types of reinforced disc. The following conclusions are obtained:

1-The existing residual stresses will cause to reduce the radial and circumferential stresses in general.

2-The crack initiates near the inner radius of a disc as a result of existing residual stresses, rather than the outer radius when neglecting residual stress.

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